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NEW YORK UNIVERSITY
Courant Institute of Mathematical Sciences
Division of Electromagnetic Research

FINAL REPORT

Basic Mathematical Investigations in Electromagnetic Wave Theory

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APRIL 1, 1959–MARCH 31, 1964

Prepared for
AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

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I. Introduction

This report is a summary of research activities pursued during the period from April 1, 1959 to March 31, 1964 by the Division of Electromagnetic Research of the Courant Institute of Mathematical Sciences at New York University, under Contract Number AF 19(604)5238 with the Air Force Cambridge Research Laboratories. The research program was a continuation of work done under Contract Numbers AF 19(122)-42 and AF 19(604)1717 with the same agency.

The object of the research was to carry out basic mathematical investigations in electromagnetic theory with emphasis on problems of diffraction, wave guides, radiation and communications. Investigations have been conducted in various areas in accordance with these broad objectives.

We classify the completed work into a number of categories, the first and largest being diffraction theory. The latter is subdivided into a number of sections as follows:

1. Diffraction at high frequencies.

   This section includes, first of all, the development, verification and application of asymptotic methods, the most important of these being based on geometrical optics. An analysis of the role played by geometrical optics as an asymptotic method in electromagnetic theory is given in EM-171 (Kline). EM-175 (Keller) constitutes a new presentation of the highly-developed and refined extension of geometrical optics called the "geometrical theory of diffraction". This method, evolved by Professor Keller during the tenure of the two previous contracts, is extended in EM-131 (Buchal and Keller) to deal with the field near singular points of the geometrical optics solution (caustics, foci, shadow boundaries, etc.) by a boundary layer method. A similar method is also used in EM-186 (Zauderer) to obtain the field near a diffracting surface at the shadow boundary. The geometrical theory of diffraction is further implemented in EM-147 (Keller and Levy) to yield higher order correction terms in the asymptotic expansion for arbitrary convex cylinders. In EM-166 (Kline), a general expression for the intensity of the geometrical optics field is obtained which applies to inhomogeneous media.

In the absence of a rigorous proof of the general validity of the geometrical theory of diffraction, it is desirable to compare the results of this method with those of other methods, both exact and asymptotic, wherever feasible; this is done in EM-147 (Keller and Levy), EM-167 (Magiros and Keller) and EM-143 (Karp and Keller), with complete agreement in every case.

In EM-188 and EM-189 (Jones), the problems of diffraction by a circular disk of an acoustic and electromagnetic wave, respectively, are treated asymptotically in another way - by obtaining a new integral formulation which leads to an asymptotic expansion at high frequencies.

Also included in this section is a report on optical diffraction theory, EM-136 (Wolf and Richards), which treats the electromagnetic field in the region of focus of certain optical systems with wide angles of aperture.

A high frequency investigation which uses the eigenfunction method, rather than an asymptotic method, is reported in EM-162 (Rubinow and Keller) in which the shift of the actual shadow boundary from the geometrical optics boundary is examined.

2. Diffraction of low frequency radiation.

Results in this area are given in EM-144 (Bazer and Brown) and EM-170 (Bazer and Hochstadt); these treat diffraction by an aperture in a plane screen using integral equations which are solved asymptotically at low frequencies. EM-158 (Rich and Karp) treats the static problem for finite conical shells, which makes it possible to treat diffraction by conical shells at low frequencies. A basic discussion of diffraction at low frequencies is given in EM-184 (Karp), where a general method is outlined for improving the usual integral equation method so as to extend its frequency range to the point where it overlaps the range of the geometrical theory of diffraction.

3. Anisotropic Surfaces.

This section of the work on diffraction is concerned with waves in the presence of modified surfaces such as a screen composed of fine parallel wires, idealized as a screen which is perfectly conducting in one direction, or a corrugated plane, which may be approximated by an anisotropic impedance boundary condition. These problems are handled, for a dipole source above an infinite plane surface in each case, in EM-180 (Karal and Karp) and EM-181 (Chu and Karp) respectively. A unidirectional conducting screen is also treated in EM-183 (Karp and Karal) for the case of a phased line source.

4. Far field expansions.

In most electromagnetic and acoustic diffraction problems the behavior of the far field scattering amplitude is of great importance. Results of investigations devoted to this aspect of the diffracted field are reported in EM-145 (Karal and Karp), EM-169 (Karp) and EM-179 (Karp) where special reference is made to the inverse scattering problem.

5. Dielectric interfaces.

This section is concerned with reflection and transmission at interfaces between homogeneous dielectric media. EM-177 (Gardner and

Keller) treats the field of a pulsed dipole in a plane interface. EM-153 and EM-154 (Kane and Karp) are concerned with three media - a dielectric half-space above two dielectric quarter spaces. In EM-140, reflection and transmission for a special class of dielectric layers with interesting properties are considered.

6. Other results in diffraction theory.

There are two reports which deal with diffraction theory but which do not fall into any of the above sections. EM-156 (Kraus and Levine) treats the problem of diffraction by an elliptic cone or plane angular sector using the eigenfunction method in a sphero-conal coordinate system. EM-135 (Noble) is also concerned with the problem of the plane angular sector; the angular eigenfunctions are studied by reducing the problem to a certain mixed boundary value problem on the surface of a sphere, and an approximate solution is obtained for the lowest eigenvalue and eigenfunction.

Our second category concerns the investigation of surface wave phenomena. In EM-159 (Kane), the excitation of surface waves on a reactive half-plane (the other half-plane being perfectly conducting) is considered. In EM-146 (Karal and Karp), the field arising when a surface wave travels along one face of a right-angled wedge and is scattered by the tip is studied. In EM-178 (Morgan and Karp), a uniqueness theorem is proved for a surface wave problem involving a mixed boundary value problem for a right-angled wedge. The excitation of surface waves is also investigated in EM-180, EM-181 and EM-183, already mentioned above in connection with diffraction by anisotropic surfaces.

Another area of investigation has been wave guide problems. In EM-132 and EM-133, Spruch and Bartram apply variational methods to obtain bounds on the equivalent network elements for obstacles in wave guides. In EM-168 (Fox and Magnus), the reflection coefficient for the basic mode in a straight two-dimensional wave guide with a discontinuity in its width is computed for small wave numbers.

The next category has to do with results concerning antennas, reported in EM-142 (Newman and Magnus) and EM-172 (Rabinowitz and Magnus); these treat the problem of determining the maximal efficiency for all possible current distributions in a linear finite antenna.

A large area of investigation concerns itself with statistical problems connected with communication theory and noise, and wave propagation in a random media. EM-152 (Silverman and Chang) is an expository article dealing with the theory of discrete information channels. In EM-151 and EM-160 (Silverman), certain types of non-Gaussian shot noise having special properties of interest are considered. EM-134 (Shapiro and Silverman) treats the problem of determining the power spectrum of a stationary random process from measurements made at discrete instants of time. Wave propagation in random media is treated in a basic report, EM-164 (Keller), wherein the two principal types of methods for solving the problems in this field are discussed, examples given and some new results obtained.

Some general mathematical results have also been obtained which are not restricted to specific problems in electromagnetic theory, and consequently are placed in a separate category. One, reported in EM-137 (Noble and Peters), presents a direct method based on the more circuitous "Wiener-Hopf technique" for solving a single integral equation of the Wiener-Hopf type. In EM-150 (Chester and Keller), asymptotic solutions of systems of linear ordinary differential equations with discontinuous coefficients are studied.

There is one miscellaneous report, EM-191, consisting of a translation by Hill of an important paper by the Russian scientist, D. M. Eidus, on the relationship between the time independent and time dependent equations of electromagnetic theory.

During the contract period, a total of 44 reports have been issued, 35 of which have been published or accepted for publication in technical journals.

The development of new talent in this important field is one of the activities supported by the contract. During the contract period, the following students obtained their doctorates, supported either wholly or in part by contract funds: R. Buchal, J. Kane, L. Levine, N. Newman, Y. M. Chen, E. Zauderer, N. Hills and M. Machover.

We have been fortunate in having a number of guests from other institutions visit us during the contract period. These, together with the meetings attended by members of the EM group, are listed at the end of this report.

We shall now describe in more detail the work completed under this contract. (A complete list of reports appears on page 27.)

II. Diffraction Theory

1. Diffraction at high frequencies.

Some of the most powerful techniques for analyzing high frequency wave propagation are to be found in geometrical optics and its extensions and refinements. The relationship between the field determined by geometrical optics and that determined by solving the appropriate boundary value problem of electromagnetic theory is that the former is an asymptotic form of the latter. A historical survey of geometrical optics with regard to this relationship, as well as an examination of the underlying theory, is presented in EM-171 (Kline).

The geometrical theory of diffraction extends geometrical optics in such a way as to account for diffraction effects, e.g. the bending of light around obstacles. This is done by introducing, along with the usual rays appearing in ordinary geometrical optics, new rays called diffracted rays. These are produced by incident rays which hit edges or corners of boundary surfaces, or which graze such surfaces. Some of these enter shadow regions and account for the light there. The initial value of the field on a diffracted ray leaving a point on a boundary is obtained by multiplying the field on the incident ray at that point by an appropriate diffraction coefficient; these coefficients are analogous to the reflection or transmission coefficients used for obtaining the field on reflected or transmitted rays at a surface. In order to obtain such a coefficient for a particular type of diffracted ray, for example, one produced by a point on a thin edge, it is necessary to solve exactly a problem having the same type of boundary singularity, e.g. the semi-infinite plane. Such problems are called canonical problems. Another way in which the geometrical theory of diffraction extends ordinary geometrical optics is to provide higher order correction terms for the field; canonical problems are required in this connection also.

Finally, another very important aspect of this method should be mentioned: it serves as a tool and provides a language for interpreting and classifying various terms of a possibly complicated asymptotic solution obtained by other methods, say by expanding an exact solution asymptotically. EM-175 (Keller) contains the most recent general treatment of the geometrical theory of diffraction and brings up to date the various developments of the theory and the list of canonical problems which have been solved.

The field predicted by geometrical optics possesses certain singularities at places such as caustics and foci, where the field becomes infinite, and at shadow boundaries, where it is discontinuous. In EM-131 (Buchal and Keller), the neighborhoods of caustics and shadow boundaries are interpreted as boundary layers, and expansions of the field are determined in these layers. These boundary layer expansions are finite at the caustics and continuous at the shadow boundaries; they thus provide an important extension of the geometrical theory of diffraction. Also, in EM-186 (Zauderer), the problem of determining the field near a diffracting surface at the shadow boundary is handled by treating the neighborhood of the point of diffraction as a boundary layer.

In EM-147 (Keller and Levy), diffraction by an elliptic cylinder is investigated as a canonical problem for the higher order corrections to the decay exponents and diffraction coefficients for arbitrary convex cylinders. The geometrical theory of diffraction thus extended is then applied to the problem of diffraction by a parabolic cylinder, and the results compared with the asymptotic expansion of the exact solution; the values are found to agree completely. The field is also determined for a hard convex cylinder of arbitrary shape asymptotically in a different way - namely, solving an integral equation by an asymptotic method; again the results of the geometrical theory of diffraction are completely verified. Further verification is obtained in EM-167 (Magiros and Keller), where the diffraction of a cylindrical wave by a perfectly conducting semi-infinite thin screen with a cylindrical tip is analyzed by three different methods; these include the geometrical theory of diffraction, the Watson transformation of the angular eigenfunction expansions, and an expansion in radial eigenfunctions. The second is the only rigorously established method. The asymptotic expansion for high frequencies obtained by this method agree completely with the results of the geometrical theory of diffraction. Also, the second and third methods yield the same result, proving that the solution can be expanded in radial eigenfunctions even though they are not complete. In EM-143 (Karp and Keller), the results of the geometrical theory of diffraction are compared with those of a second method, also asymptotic, which is applicable to problems involving multiple diffraction - "the self-consistent field method"; the problem considered is that of diffraction by a slit in a hard screen. The results agree at distances greater than several wave lengths from the edges of the screen, thus lending support to both methods. The geometrical theory of diffraction is also applied in this paper to the problem of diffraction by a circular aperture in a hard screen.

Another asymptotic method at high frequencies is developed in EM-188 (Jones). Here the integral equation for the acoustic field on a circular disk in an axisymmetric sound wave is converted into an integral equation of the second kind which is especially useful at high frequencies; a method is then devised for obtaining an asymptotic expansion at high frequencies. This procedure is generalized to the case of electromagnetic waves in EM-189 (Jones).

In EM-162 (Rubinow and Keller), the problem of diffraction by a cylinder is studied to determine the displacement of the actual shadow boundary from the geometrical shadow boundary. The investigation was undertaken to clear up a discrepancy which exists between the results of Artmann and Rice in the asymptotic expression at high frequencies for the boundary shift for a hard cylinder. The shift obtained through the use of a Watson transformation of the angular eigenfunction expansion verifies Rice's results.

2. Diffraction of low frequency radiation.

The problems of the diffraction of scalar waves by a circular aperture in a soft and hard plane screen for axially symmetric excitation are treated in EM-144 (Bazer and Brown). New integral representations of the solutions are presented which involve unknown functions f_1 and f_2 for the soft and hard screens respectively. These f's turn out to be solutions of Fredholm-type integral equations of the second kind whose kernels are small when ka is small. (k is the wave number and a is the radius of the aperture.) For small ka, these integral equations lead to approximate expressions for the f's which can be employed to calculate the aperture fields, the far field, the transmission coefficients and the edge behaviors. EM-170 (Bazer and Hochstadt) presents the generalization for non-axially-symmetric excitations.

In EM-158 (Rich and Karp), the problem of incompressible fluid flow in the presence of a finite conical shell for the axially symmetric case is solved.

When interpreted as a static result in electromagnetic theory, this solution makes it possible to solve the problem of diffraction by a conical shell asymptotically for low frequencies by means of the "Raleigh static method".

In EM-184 (Karp), the general problem of diffraction at low frequencies is considered with a view to improving the frequency range of the integral equation approach usually employed in this connection. The original integral equation, say the Green's function integral equation, is converted by means of the static Green's function to a new Fredholm equation of the second kind. The solution of the latter can be constructed by the Fredholm integral method as a quotient of two entire functions; these can be represented by two everywhere convergent power series whose coefficients are simply expressed in terms of the coefficients of the usual low frequency (Neumann) expansion. One can thus construct a sequence of fractions which converges at all frequencies, and is asymptotic at low frequencies.

3. Anisotropic surfaces.

The propagation and diffraction of electromagnetic waves in the presence of anisotropic surfaces is of considerable practical importance. Two reports, EM-180 (Karal and Karp) and EM-183 (Karp and Karal), deal with screens consisting of an infinite number of straight, perfectly conducting parallel wires. Such screens are idealized as being conducting in only one direction, the boundary conditions being prescribed as follows: (1) the electric field component parallel to the wires is zero on both sides of the screen, (2) the component of the electric field normal to the wires is continuous across the screen, and (3) the component of the magnetic field parallel to the wires is continuous across the screen. In EM-180, the source of the electromagnetic field is an electric dipole source located above the screen and oriented in a direction perpendicular to the wire elements. In EM-183, the source is a phased line source similarly located and oriented; it consists of a periodic line current with an electric charge distributed along its length. In both cases the solution is obtained exactly. (In EM-180 it is expressed in a simple elementary form.) Expressions are given for the radiated far field components and the radiated power. Also, the amplitudes of the surface waves which are excited are determined, together with the power propagated by them. In EM-180 these waves are not surface waves in the usual sense: they decay inversely as the square of the distance from the screen and hence are not as tightly bound to the surface as surface waves of the usual type which decay exponentially.

EM-181 (Chu and Karp) deals with an infinite corrugated plane which is approximated by an isotropic impedance boundary condition. This is of the form

$$E_x = ZH_z = (R - iX)H_z$$

$$E_z = 0$$

on the plane $y = 0$; the time dependence is of the form $e^{-i\omega t}$ and Z, R and X are the impedance, resistance and reactance of the surface. In the case considered, $R = 0$ and $X > 0$, so that the surface supports surface waves. The fields produced by a magnetic dipole are studied. The exact solution is obtained, and the radiated far field and surface wave far field evaluated.

4. Far field expansions.

Radiative solutions of the three-dimensional Helmholtz equation $\Delta u + k^2 u = 0$ are known to be representable in the form

$$u = r^{-1}\exp(ikr)f(1/r,\theta,\phi) \quad \text{with} \quad f(1/r,\theta,\phi) = f_0(\theta) + \sum_{n=1}^{\infty} f_n(\theta,0)r^{-n},$$

where the series is both asymptotic and convergent. In fact $f(1/r,\theta,\phi)$ is known to be an analytic function of $1/r$ for $|r| > a$ provided all the singularities of u are contained in the sphere of radius a. On the other hand, in two dimensions, the corresponding farfield expansion

$$u = r^{-1/2} e^{ikr} \left[f_o(\theta) + \sum_{n=1}^{\infty} f_n(\theta) r^{-n} \right]$$

is asymptotic, but is not convergent. In EM-169 (Karp), this defect is remedied by showing that any radiative solution of the two-dimensional Helmholtz equations can be written in the form $u = H_o^{(1)}(kr)\, E(\frac{1}{r},\theta) + H_1^{(1)}(kr)\, O(\frac{1}{r},\theta)$, where the functions E and O are analytic functions of $\frac{1}{r}$, for $|r| > a$, and are of course expressible as convergent power series in $\frac{1}{r}$. Here a is the radius of the smallest circle containing the singularities of u. Furthermore, a method is presented for deducing $E(0,\theta)$ and $O(0,\theta)$ from the customary farfield datum, namely $f_o(\theta)$, and vice versa.

In EM-145 (Karal and Karp), a new simple direct method for the determination of the diffracted farfield for wedges with impedance boundary conditions is presented. The wedge angle is $\pi/2n$, where n is an integer. The excitation may be either an incident plane wave or a line source. The method is described in detail for the case of an absorbing wedge with a line source located at the tip. Several other examples are given to illustrate the method.

Some new properties of the far field scattering amplitude for plane wave excitation are presented in EM-179 (Karp). Two dimensional scalar scattering is treated. The scattering amplitude is denoted by $f(\theta,\theta_o)$, where θ_o is the angle of incidence of the plane wave and θ is the angle of observation of the far diffracted field; $f(\theta,\theta_o)$ is always an analytic function of θ. The problem of inverse diffraction theory is to determine the boundary surface given $f(\theta,\theta_o)$ for a fixed wave number, and the boundary condition. In this paper some results which relate f to the boundary surface under special circumstances are proved. One such result involves the case where $f(\theta,\theta\ ) = f(\theta - \theta_o)$ in the case of a soft obstacle. Such a scattering amplitude obviously arises when the scatterer is a circle. It is proved that this is the only possible boundary.

5. Dielectric interfaces.

Most electromagnetic problems involving more than one dielectric medium are not capable of exact solution. An example of such a problem which is of considerable importance in applications is that of a dielectric half space above two different dielectric quarter spaces. In EM-153 and EM-154 (Kane and Karp) an effective technique is developed for solving this problem approximately when one is interested in obtaining the field in the upper half space. This is done by replacing the left and right interfaces forming the boundary of the upper half space by two linear boundary conditions. The resulting mixed boundary value problem in the upper half space is solvable by the Wiener-Hopf procedure if the boundary conditions have constant coefficients. The solution of the mixed boundary value problem will approximate the true field more or less closely depending on how well one can approximate the effect of a dielectric interface by a linear boundary condition of the type indicated. In Part I, i.e. EM-153, the formulation and justification of the boundary condition is discussed; in Part II, EM-154, the above problem is solved.

In EM-177 (Gardner and Keller), the problem of a pulsed vertical electric dipole in the plane interface between two semi-infinite homogeneous dielectric media is considered. If the dipole is switched on at $t = 0$, its moment being zero for $t \leq 0$ and constant for $t > 0$, an electromagnetic field will radiate outward from the dipole. This field is determined by finding the corresponding two dimensional field due to a line dipole and then making use of a new representation of the field of a point source as a superposition of the fields of line sources. An elementary expression for the field of a line dipole is found using the conical flow transformatio which is well known in the theory of linearized flow.

In EM-140 (Karp) the reflection and transmission of electromagnetic waves by a special class of curved dielectric layers are considered. The layers in question are bounded by two non-intersecting confocal parabolas, and between these boundaries, the dielectric constant, which may be complex, has a special form of space dependence. It is found that layers of this type have striking properties. If a plane wave is normally incident on such a layer, from the convex side, then the reflected field can be expressed as a product of two factors. The first factor is the field which would be reflected by a conducting parabola coincident with the surface of the layer. The second factor has no space dependence; it depends only upon the spacing of the shell boundaries, the frequency and the special choice of dielectric constant. The transmitted wave has an especially simple form. It is a plane wave. The configuration thus has properties similar to those of a plane dielectric slab. In fact, the plane slab is a limiting case of the dielectric layers described.

6. Other results in diffraction theory.

In EM-156 (Kraus and Levine), the boundary value problems corresponding to a spherical wave incident on an acoustically soft or hard core, or plane angular sector, (which is a degenerate elliptic cone), are solved. The method of separation of variables in a sphero-conal coordinate system is used. The special functions which arise are simply periodic and non-periodic solutions of the Lamé differential equation, products of which form the angular eigenfunctions of the problem. The problem of the plane angular sector in particular is important for the geometrical theory of diffraction, since it is the simplest case of a plane screen with a sharp corner; if a suitable asymptotic expansion of the solution at high frequencies could be found, this problem would constitute a canonical problem for determining the vertex diffraction coefficients of such a corner.

In EM-135, the potential and charge distribution near the tip of a plane angular sector are calculated by reducing the problem to the solution of a mixed boundary value problem on the surface of a sphere. A method is developed for finding the lowest eigenvalue μ and corresponding eigenfunction. (The angular eigenfunctions and eigenvalues of this problem are identical to those of the diffraction problem for a plane angular sector.) If s is the distance from the tip of the sector, then for small s the potential varies as r^{μ}. Numerical results are given for μ as a function of the sector angle.

III. Surface Waves

This category, of course, overlaps the category on diffraction theory, particularly that section of the latter concerned with anisotropic surfaces. All the surfaces discussed therein support surface waves, as was brought out in our survey of EM-180, EM-181 and EM-183.

In EM-159 (Kane), the following two-dimensional problem is considered: An arbitrary antenna is located at the origin which launches surface waves on a reactive half-plane $y = 0$, $x < 0$. It is assumed that the other half-plane, $y = 0$, $x > 0$ is occupied by a perfect conductor. The antenna is approximated by a distribution of multipoles easily found from the antenna's radiation pattern. With this approximation, simple algebraic expressions are obtained for the ratio of the power delivered to the surface waves as compared to the power in the radiated cylindrical field.

In EM-146, Karal and Karp study the electromagnetic field that arises when a surface wave travels along the front face of a right angled wedge and is scattered by the tip. An impedance type boundary condition is prescribed on the front face of the wedge and a simple boundary condition (that for perfect reflection) is prescribed on the other. The impedance boundary condition is such that surface waves are generated. An exact mathematical solution in elementary form is presented for this problem and the amplitude of the reflected surface wave found. Also, a simple representation for the radiated far field amplitude is given. The important special cases of small and large reactance are discussed. For large reactance the wave is closely bound to the surface, which may then be regarded as a surface-wave-guide. In this limit there is complete reflection of the surface wave, with a change in phase of $\pi/3$, for the geometry considered.

In EM-178 (Morgan and Karp) a uniqueness theorem is proved for scalar diffraction by a right-angled wedge where a Neumann boundary condition is prescribed on one side and an "impedance" boundary condition on the other side. In the latter, the surface impedance is such as to support surface waves. A two dimensional radiation condition is prescribed for the radiative part of the solution $u_{rad} = u - u_{incident} - u_{reflected}$.

IV. Wave Guide Problems

In EM-132 and EM-133, Spruch and Bartram use a variational method developed by Kato (for problems in quantum mechanics) to obtain upper and lower bounds for the phase shifts associated with dielectric obstacles in lossless wave guides. Since the reactances of the equivalent T-network are related in a simple way to these phase shifts, these results yield lower and upper bounds for these reactances. The general theory is developed in EM-132, and the bounds are computed for dielectric obstacles of various shapes in EM-133.

The perturbation method is used in EM-168 (Fox and Magnus), with the electrostatic case as the unperturbed case, to compute for small wave numbers the reflection coefficient for the basic work in a straight, two-dimensional wave guide with a discontinuity in its width. Essential features of the treatment are:

(1) the use of conformal mapping to solve the electrostatic problem,

(2) the use of physically unreal modes, and

(3) the inversion of an infinite matrix.

V. Antenna Problems

The problem of determining the maximal efficiency of an antenna is investigated in EM-142 (Newman and Magnus). The efficiency of an antenna is defined as the ratio of the radiated energy $\mathcal{E}$ to the heat loss $\mathcal{H}$. For a linear, finite antenna of length 2ℓ, this efficiency can be shown to have a least upper bound B for all possible current distributions and for all ℓ. B is determined explicitly. For a fixed small value of ℓ, the maximal efficiency is computed up to an error term of order ℓ^3. If the efficiency of an antenna is defined as the ratio of the energy $\mathcal{E}^*$ radiated into a half-space, to the total heat loss $\mathcal{H}$, similar but less complete results can be derived. Finally, it is shown that the problem of maximizing $\mathcal{E}^*/\mathcal{E}$ does not have a solution. There exists no realizable current distribution on the antenna which maximizes this quotient.

In EM-172 (Rabinowitz and Magnus) the maximal efficiency is computed numerically from the results of EM-142 by solving the Fredholm integral equations, derived in the earlier report, using a method of successive polynomial approximation.

VI. Statistical Problems

A variety of reports are concerned with information theory and noise. The expository article EM-152 (Silverman and Chang) discusses the following topics in the theory of discrete information channels:

(1) information sources and information rate,
(2) channels and mutual information rate
(3) channel capacity
(4) the general binary channel
(5) channels with fading
(6) cascaded channels
(7) channels with memory.

In EM-151 (Silverman), a family of widely different types of shot noise is constructed, all having the same power spectrum; this illustrates the inadequacy of the power spectrum as a characterization of random processes. EM-160 (Silverman) is concerned with a non-Gaussian shot noise which, like the turbulent velocity field, is univariate normal, but not bi-variate normal.

In EM-134, Shapiro and Silverman consider the question of when the power spectrum of a stationary random process is uniquely determined by the values of the process measured in various ways at discrete instants of time. In the case of equi-spaced sampling, it is known that the power spectrum is not uniquely determined by the sampled data, i.e. aliasing is present. After a review of the equi-spaced case, the case where the time markers are not equi-spaced, but are subject to jitter, is examined, and it is found that in general aliasing persists. The authors then consider the case where the time markers are generated by an additive scheme, whereby each sampling time is derived from the previous one by the addition of an independent random variable, with characteristic function $\phi(s)$. It is found that additive random sampling is alias-free if $\phi(s)$ is one-to-one on the real axis, but not alias-free if $\phi(s)$ takes the same value at two distinct points of the open upper half-plane. Various alias-free sampling methods are exhibited, notably Poisson sampling.

In the basic EM-164 (Keller), "wave propagation" and "random media" are defined, and the nature of the mathematical problems arising in wave propagation in random media is described. The two principal types of methods for solving these problems - "honest" and "dishonest" methods - are explained. These methods are first illustrated by considering the geometrical optics of a random medium by one method of each type. Some new results are obtained by an honest method and some errors in a previous work are pointed out. Comparison is made between the results of the two methods and the reasons why they disagree are explained. As a second illustration of an honest method, an analysis of the reduced wave equation in a random medium is presented. Some known results are obtained in a new way which is simpler than the usual one and which appears to be capable of yielding further results.

VII. Mathematical Results

In EM-137 (Noble and Peters) it is shown that an explicit inversion formula exists for integral equations of the form

$$\int_0^\infty k(x - \xi)\, f(\xi)d\xi = g(x) , \qquad (0 \leq x \leq \infty)$$

where g(x) is arbitrary. The solution depends on two functions derived from the Wiener-Hopf factorization; once these are found, the solution is obtainable through real integrations.

In EM-150 (Chester and Keller), a system of linear first order ordinary differential equations is considered in which the coefficient matrix depends upon a parameter k. The asymptotic form of the solutions of this system are determined for large values of k when the coefficient matrix or some of its derivatives have finite jump discontinuities. The asymptotic form obtained contains terms beyond those previously known. These new terms have discontinuous coefficients, whereas the previously known terms are continuous.

The results are applied to the one dimensional propagation of time-harmonic waves. It is shown that in this case the reflection coefficient is proportional to k^{-J} if the J-th derivative of the index of refraction is discontinuous and the lower order derivates are continuous. Here $k = 2\pi/\lambda$ is the propagation constant and λ the wavelength in a medium of unit refractive index. This result should be compared with the fact that the reflection coefficient vanishes faster than any negative power of k if all derivatives of the index of refraction are continuous.

VIII. Translation of a paper on basic theory

In many practical problems of electromagnetic theory the sources producing the electromagnetic field have a sinusoidal time dependence.

It is then assumed that the resulting electromagnetic field also has this time dependence and the resulting problem then becomes a simpler problem which is essentially time independent.

It has been long assumed that this method of analysis yields the steady state solution. However, it was not until recent time that attempts were made to prove this fact. Because of the continuing interest in this problem the Division of Electromagnetic Research felt it would be worthwhile to publish a translation, EM-191 (Hill), of a paper by the Soviet scientist D. M. Eidus. This paper by Eidus treats the problem in the greatest generality which has yet been undertaken. Since he considers a very general equation and boundary his results contain and extend the results of other workers in this field.

A parallel problem also considered by Eidus in this paper concerns the principle of limiting absorption. In practical problems this principle states that if a boundary value problem is solved for a medium which is slightly conducting and then the conductivity is allowed to approach zero the resulting limit of the solution is the solution of the same boundary value problem in a non-conducting medium. This principle is important since it is often simpler in the mathematical analysis to introduce an artificial conductivity in order that a solution may be more readily analyzed.

VISITORS TO THE GROUP AND MEETINGS ATTENDED

May 1, 1959	Dr. Werner Gerbes, of the AFCRC, discussed contract matters with Dr. Sidney Borowitz. Prof. Bernard Friedman, of the University of California, visited the group to discuss problems of diffraction; and Dr. Victor Twersky, of Sylvania Electric Products Corp., Mountain View, California, problems related to electromagnetics.
May 7, 1959	Prof. V. Fock, of Leningrad University, Russia, visited the group for the purpose of discussing problems of electromagnetics and quantum mechanics.
May 7, 1959	The following papers were presented at the URSI meeting in Washington, D. C.: "Multiple Diffraction by An Aperture in a Hard Screen" -- J. B. Keller and S. N. Karp "Diffraction by a Prolate Spheroid" -- B. Levy and J. B. Keller "Excitation and Propagation of Surface Waves" -- F. C. Karal and J. B. Keller.
May [illegible], 1959	Prof. V. M. Papadopoulos, of Brown University, presented a talk entitled "Diffraction and Refraction of Plane Pulses."
June 5, 1959	Dr. B. A. Auld, of Stanford University, discussed problems of non-linear wave propagation in microwave systems with Drs. Keller, Karp, Karal and Borowitz.
June 14-19, 1959	The following papers were presented at the URSI meeting in Toronto, Canada: "The Field of a Pulsed Dipole in an Interface" -- J. B. Keller and C. S. Gardner "Diffraction by an Inhomogeneous Sphere" -- B. Levy and J. B. Keller "On the Behavior of Electromagnetic Waves in the Presence of Wedges with Impedance Boundary Conditions" -- S. N. Karp and F. C. Karal "High Frequency Diffraction by Strips and Discs" -- B. Noble "Diffraction by a Circular Aperture" -- J. Bazer and A. Brown "Fields in the Neighborhood of a Caustic" -- I. Kay

Aug 26-31, 1959	Prof. J. B. Keller attended the American Physical Society meeting in Honolulu, Hawaii.
Sept. 8, 1959	Dr. Josef Meixner, of Aachen, Germany, visited the group.
Sept. 14, 1959	Assoc. Prof. Alfred Leitner of Michigan State University, visited the group.
Sept., 1959	Dr. Horst Neimeyer, from Aachen, Germany, visited the group for one year.
Oct. 9, 1959	Dr. Ben Noble, of the Royal College of Science and Technology in Glasgow, returned to Scotland after having spent a year with the group.
Oct. 19, 1959	Julius Kane presented a talk, "An Approximate Boundary Condition to Replace Dielectric Media", at the URSI Meeting in San Diego, Calif.
Nov. 12, 1959	Dr. Emil Wolf, of the University of Rochester, and Dr. Samuel Silver, of the University of California, visited the group.
Nov. 25, 1959	Dr. Morris Kline presented a colloquium and discussed contract matters with scientists at the AFCRC, Bedford, Mass.
Dec., 1959	Mr. Robert Shore, of the AFCRC, visited Drs. Kline, Borowitz and Karp to discuss contract work.
Jan. 27-30, 1960	Dr. M. Kline attended the American Math. Society's meeting in Chicago.
Jan. 26, 1960	Drs. E. Mostue and J. Kielson,of Sylvania, Waltham, Mass., visited Drs. Kline and Borowitz.
Feb. 19, 1960	Dr. Makato Itoh, of Brooklyn Polytechnic Institute, presented a talk entitled "Electromagnetic Function Theory".
March 4, 1960	Dr. Hermann Poeverlein, of the AFCRC, presented a talk entitled "Theory of Ionospheric Reflections".
March 10, 1960	Prof. Robert Thrall, of the University of Michigan, visited Dr. Kline.
March 22, 1960	Dr. K. M. Siegal, of the University of Michigan, presented a talk, "Theory of Non-linear Modeling".
March 31, 1960	Dr. Kline presented a talk at the Willow Run Laboratories on the subject of current electromagnetic wave problems.

April 8, 1960	Prof. Claus Müller, of the Technische Hochschule in Aachen, Germany, visited the group and presented a talk, "Asymptotic Solutions of Two-Dimensional Diffraction Problems".
April 14-16, 1960	Drs. Kline, Magnus and Keller attended the American Math. Society's meeting and Dr. Keller spoke on the subject "Wave Propagation in Random Media".
April 29, 1960	Prof. Calvin Wilcox, of the University of Wisconsin, spoke on "Scattering of Arbitrary Acoustic Fields by Spheres".
May 1-3, 1960	Drs. Kline, Karp, Karal, Levy and Keller attended the URSI meeting in Washington, D. C., where Dr. Karp presented a paper, "Scattering of a Surface Wave Discontinuity in the Surface Reactance on a Right-Angled Wedge".
May 13, 1960	Dr. E. Wolf, of the Institute of Optics at the University of Rochester, presented a paper, "Some Recent Research on Coherence and Polarization Properties of Light Waves".
May 16, 1960	Profs. J. J. Seidel and G. W. Veltkamp, of Eindhoven, Holland, visited the group.
June 8-11, 1960	Dr. Keller attended the International Conference on Partial Differential Equations at Madison, Wisconsin.
June 21-24, 1960	Dr. B. Levy presented a paper entitled "Ray Methods in High Frequency Approximations" at the UCLA Symposium.
Aug. 21-Sept. 1, 1960	Dr. M. Kline attended the American Math. Society's meeting at the University of Michigan.
Oct. 7, 1960	Dr. S. H. Northover, Carleton College, visited the group and presented a talk: "Diffraction by a Finite Cone".
Oct. 13, 1960	Dr. J. B. Keller attended the Boston meeting of the Optical Society of America and presented a paper: "Geometrical Theory of Diffraction".
Oct. 14, 1960	Prof. M. Papadopoulos, Brown University, spoke to the group on the subject: "Problems of Potential Theory Associated with Pulse Diffraction and Refraction".
Nov. 11, 1960	Dr. Hans Wilhelmsson, Princeton University, presented a talk to the group: "Diffraction of Electromagnetic Waves by Electron Beams".

Dec. 8, 1960	Captain John MacCullum, Wright Field, visited Drs. Kline, Karp and Karal.
Dec. 9, 1960	Dr. R. I. Primich, of the Defense Research Board, Ottawa, Canada, spoke to the group on the subject: "Experimental Studies of Diffraction and Plasma Theory".
Dec. 12-14, 1960	Drs. J. B. Keller and S. Karp attended the URSI meeting at Boulder, Colorado. Dr. Keller presented a talk entitled, "The Shift of the Shadow Boundary and the Scattering Cross Section of an Opaque Object".
Dec. 19, 1960	Dr. Werner Gerbes, of AFCRL, visited Dr. Kline to discuss contract problems and the course of future research.
Jan. 16, 1961	Dr. F. Zucker, of AFCRL, visited Dr. Kline to discuss contract matters.
Jan. 23-27, 1961	Profs. Kline and Keller attended the American Math. Society's meeting in Washington, D. C.
Feb. 16-17, 1961	Dr. J. Meixner, of the Technische Hochscule, Aachen, Germany, and Dr. Emil Wolf, of the Institute of Optics, University of Rochester, visited the group.
Feb. 27, 1961	Dr. James Wait, of Boulder, Colorado, visited the group to discuss the "Symposium on Electromagnetic Theory and Antennas" held in Copenhagen, June, 1962.
March 3, 1961	Dr. G. Goubau, of Fort Monmouth, N. J., presented a talk, "Electromagnetic Wave Beams".
March 10, 1961	Dr. H. Poeverlein, of AFCRL, presented a talk: "Propagation of Electromagnetic Waves in a Plasma with Strong Magnetic Field".
March 24, 1961	Dr. Farouk Odeh, of IBM Research Center, Yorktown Heights, presented a talk: "Existence and Uniqueness of the Solution of Shrödinger's Equation".
April 10, 1961	Dr. S. N. Karp presented a paper, "Some Properties of Farfield Amplitudes" at the University of Wisconsin.
May 3, 1961	Dr. S. N. Karp and N. Hills presented a joint paper entitled "Diffraction by a Semi-infinite Grating" at the URSI Washington, D. C. meeting.
May 3, 1961	Dr. Robert Millar presented a paper entitled, "Plane Wave Spectra in Multiple Scattering Theory" at the URSI meeting in Washington, D. C. This meeting was also attended by Drs. M. Kline, Erik Hansen and J. Keller.

Oct. 21, 1961	Prof. S. N. Karp attended a technical meeting of the URSI in Austin, Texas.
Nov. 17, 1961	Dr. George B. Field of Princeton University Observatory visited the group and addressed the Electromagnetic Seminar.
January 21, 1962	Prof. S. N. Karp attended the American Math. Society meeting in Cincinnati, Ohio.
January 26, 1962	Prof. J. B. Keller attended the conference for the selection of papers for the Electromagnetic Conference in Copenhagen, June, 1962.
Feb. 23, 1962	Prof. L. B. Felsen, Brooklyn Polytechnic Institute, spoke on, "Radiation Problems in Uniaxially Anistropic Regions".
March 6, 1962	Prof. S. N. Karp presented a talk at the Electrical Engineering Department, N. Y. U., Heights: "A Surface Wave Incident Obliquely on a Unidirectionally Conducting Half-Plane".
March 23, 1962	Prof. S. A. Bowhill, Penn. State University, spoke on: "Recent Developments in Ionospheric Theory".
March 30, 1962	Prof. E. Wolf, University of Rochester, spoke to the group: "A Generalization of the Maggi-Rubinowicz Theory of the Boundary Diffraction Wave".
May 4, 1962	Dr. Yoshio Hayashi, of the University of Wisconsin, presented a talk: "Approximate Boundary Conditions on a Good Conductor".
June 25-30, 1962	Drs. Karp, Keller, Karal and Kay attended the Copenhagen Symposium on Electromagnetic Theory and Antennas, and Drs. Karp and Karal presented a paper: "Propagation of Electromagnetic Waves along Unidirectionally Conducting Screens".
April 19, 1963	Dr. Herman Poeverlein, from AFCRL, visited the Division of Electromagnetic Research.
Oct. 8, 1963	Dr. Morris Kline lectured at the AFCRL. The subject of his talk was: "Strategy and Logistics".
Oct. 15-18, 1963	Dr. S. N. Karp attended the Fall URSI meeting in Ottawa, Canada.

Dec. 11, 1963	Dr. Robert M. Lewis presented a talk at the AFCRL on the subject: "Asymptotic Methods for Partial Differential Equations".
Jan. 23-27, 1964	Dr. M. Kline attended the national meeting of the American Math. Soc. at the University of Miami.

Long-term Visitors

Dr. D. S. Jones, Dept. of Mathematics, Keele, Staffordshire, England, was a Temporary Member with the group from August, 1962 - July, 1963.

Dr. H. M. Nussenzveig, Research Scientist, of Centro Brasileiro de Pesquisas Fisicas, Brazil, worked with the group, 1963-1964.

Dr. Kane S. Yee, of Berkeley, California, worked with the group as Associate Research Scientist from September, 1963 - January, 1964.

Complete List of Reports and Publications

The following is a complete list of reports and publications of work done under Contract AF 19(604)5238 by the Division of Electromagnetic Research of the Courant Institute of Mathematical Sciences.

EM-131	R. Buchal J. B. Keller	Boundary Layer Problems in Diffraction Theory Published - Comm. Pure Appl. Math., 13, No. 1, 85 (1960).	Apr., '59
EM-132	L. Spruch R. Bartram	Bounds on the Elements of the Equivalent Network for Scattering in Waveguides. I. Theory Published - JAP, 31, No. 5, 905 (1960).	Apr., '59
EM-133	L. Spruch R. Bartram	Bounds on the Elements of the Equivalent Network for Scattering in Waveguides II. Application to Dielectric Obstacles Published - JAP, 31, No. 8, 913 (1960).	Apr., '59
EM-134	H. S. Shapiro R. A. Silverman	Alias-Free Sampling of Random Noise Published - J. SIAM, 8, No. 2, 225 (1960).	June, '59
EM-135	B. Noble	The Potential and Charge Distribution Near the Tip of a Flat Angular Sector	July, '59
EM-136	E. Wolf B. Richards	Structure of the Electromagnetic Field in the Image Region of an Aplanatic Optical System Published - Proc. Royal Soc. (London), 253, 358 (1959).	July, '59
EM-137	B. Noble A. S. Peters	A Multiplying Factor Method for the Solution of Wiener-Hopf Integral Equations	July, 59
EM-140	S. N. Karp	Reflection and Transmission by a Class of Curved Dielectric Layers Accepted - J. of Res., Nat. Bur. Stand.	Aug., '59

EM-142	N. Newman W. Magnus	Maximal Gains on Antennas	Sept., '59
EM-143	J. B. Keller S. N. Karp	Multiple Diffraction by an Aperture in a Hard Screen Published - Optica Acta, 8, No. 1, 61 (1961).	Jan., '60
EM-144	J. Bazer A. Brown	Diffraction of Scalar Waves by a Circular Aperture Published - IRE Trans. on Antennas and Prop., AP-7, S12 (1959).	Aug., '59
EM-145	S. Karp F. C. Karal	A New Method for the Determination of Far Fields with Applications to the Problem of Radiation of a Line Source at the Tip of an Absorbing Wedge Published - IRE Trans. on Antennas and Prop., AP-7, S91 (1959).	Sept., '59
EM-146	F. C. Karal S. N. Karp	Scattering of Surface Waves by a Right Angled Wedge Published - Comm. Pure Appl. Math., 14, No. 1, 35 (1961).	Feb., '60
EM-147	J. B. Keller B. Levy	Decay Exponents and Diffraction Coefficients for Surface Waves on Surfaces of Non-Constant Curvature Published - IRE, PGAP, AP-7, S52 (1959).	Oct., '59
EM-150	C. Chester J. B. Keller	Asymptotic Solution of Systems of Linear Ordinary Equations with Discontinuous Coefficients Published - J. Math. and Mech. Anal., 10, 557 (1961).	Dec., '59
EM-151	R. A. Silverman	An Isospectral Family of Random Processes Published - IRE PGIT, 6, No. 4, 485 (1960).	Jan., '60

EM-152	R. A. Silverman S. H. Chang	Topics in the Theory of Discrete Information Channels	Mar., '60
EM-153	J. Kane S. N. Karp	An Accurate Boundary Condition to Replace Transition Conditions at Dielectric-Dielectric Interfaces Accepted - J. of Research, Nat'l. Bureau of Standards	Apr., '60
EM-154	J. Kane S. N. Karp	Radio Propagation Past a Pair of Dielectric Interfaces	May, '60
EM-156	L. Kraus L. Levine	Diffraction by an Elliptic Cone Published - Comm. Pure Appl. Math., 14, No. 1, 49 (1961).	Mar., '60
EM-158	E. Rich S. N. Karp	The Virtual Mass of a Finite Conical Shell.	May, '60
EM-159	J. Kane	Surface Waves on a Reactive Half Plane Published - Transactions IRE-PGAP, AP-8, No. 5, (1960).	Apr., '60
EM-160	R. A. Silverman	A Note on the Local Structure of Shot Noise Published - IRE Trans. PGIT, IT-6, No. 5, 548 (1960).	May, '60
EM-162	S. Rubinow J. B. Keller	The Shift of the Shadow Boundary and the Scattering Cross Section of an Opaque Object Published - JAP, 32, 814 (1961).	Sept., '60
EM-164	J. B. Keller	Wave Propagation in Random Media Published - Proc. of Symp. in Appl. Math., "Hydrodynamic Instability," AMS, Vol. 13, 1962.	Oct., '60

EM-166	M. Kline	A Note on the Expansion Coefficient of Geometrical Optics Published - Comm. Pure Appl. Math., 14, No. 3, 473 (1961).	May, '61
EM-167	J. B. Keller D. Magiros	Diffraction by a Semi-infinite Screen with a Rounded End Published - Comm. Pure Appl. Math., 14, No. 3, 457 (1961).	Mar., '61
EM-168	D. Fox W. Magnus	Perturbation Methods in a Problem of Waveguide Theory Published - J. of Res., Nat. Bur. Stand. D. Radio Propagation 670 No. 2 Mar-Apr (1963).	Mar., '61
EM-169	S. N. Karp	A Convergent 'Farfield' Expansion for Two-dimensional Radiation Functions Published - Comm. Pure Appl. Math., 14, No. 3, 427 (1961).	Mar., '61
EM-170	J. Bazer H. Hochstadt	Diffraction by a Circular Aperture. II. Published - Comm. Pure Appl. Math., 15 No. 1, 1 (1962).	Apr., '61
EM-171	M. Kline	Electromagnetic Theory and Geometrical Optics Published - U. of Wisconsin 1961 Symp., "EM Waves" (R.E. Langer, Ed.), p. 3.	Nov., '61
EM-172	P. Rabinowitz W. Magnus	Numerical Data Concerning the Effectiveness of Antennas.	Apr., '62
EM-175	J. B. Keller	Geometrical Theory of Diffraction Published - J. Opt. Soc. of Amer., 52, No. 2, 116 (1962).	Apr., '62
EM-177	C. Gardner J. B. Keller	The Field of a Pulsed Dipole in an Interface Published - Comm. Pure Appl. Math., 15 No. 2, 99 (1962).	Dec., '62

EM-178	R. Morgan S. N. Karp	Uniqueness Theorem for a Surface Wave in Electromagnetic Diffraction Theory Published - Comm. Pure Appl. Math. 16, No. 1, 1963	Nov. '62
EM-179	S. N. Karp	Far Field Amplitudes and Inverse Diffraction Theory Published - U. of Wisconsin 1961 Symp., "EM Waves" (R. E.Langer, Ed.) p. 291.	Sept. '62
EM-180	F. Karal S. N. Karp	Propagation of Electromagnetic Waves Along Unidirectionally Conducting Screens Published - 1962 Copenhagen Symposium.	Nov. '63
EM-181	S. N. Karp T. S. Chu	The Field of a Dipole above an Infinite Corrugated Plane Published - Quart. Appl. Math. 21, No. 4 257-268, Jan. (1964).	Feb. '63
EM-183	S. N. Karp F. C. Karal	Excitation of Surface Waves on a Unidirectionally Conducting Screen by a Phased Line Source Accepted - Transactions IRE-PTGAP Jan., (1964).	'63
EM-184	S. N. Karp	Systematic Improvement of Quasistatic Calculations Published - Copenhagen Symposium June (1962)	'63
EM-186	E. Zauderer	Wave Propagation Around a Convex Cylinder Accepted - J. of Math. and Mechanics	'63
EM-188	D. S. Jones	Diff. at High Frequencies by a Circular Disc Accepted - Cambridge Philosophical Society	'63

EM-189	D. S. Jones	Diff. of a High-Frequency Plane Electromagnetic Wave by a Perfectly Conducting Circular Disc	'63
EM-191	Clyde D. Hill	On the Principle of Limiting Absorption (Trans. from the Russian: D. M. Eidud)	'63

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